

Polarization observables in the $e^+e^- \rightarrow \bar{\Lambda}\Lambda$ reaction

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(Dated: February 9, 2016)

Abstract

Cross-section, vector-polarization, and tensor-polarization distributions are calculated for the reactions $e^+e^- \rightarrow \bar{p}p$ and $e^+e^- \rightarrow \bar{\Lambda}\Lambda$. Each reaction requires six characteristic functions that are bilinear in the, possibly complex, electromagnetic form factors, denoted $G_E(P^2)$ and $G_M(P^2)$, of p and Λ . For the hyperon reaction also the joint-decay distributions of Λ and $\bar{\Lambda}$ are calculated. Their knowledge allow a complete determination of the hyperon electromagnetic form factors, without measuring hyperon spins. We explain how this is done in practice. For some tensor-polarization components our results are in conflict with previously repeatedly published distributions.

PACS numbers: 13.30.Eg, 13.40.Gp, 13.66.Bc, 13.88.+e, 14.20.Jn

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I. INTRODUCTION

The cross-section distributions of the reaction $e^-e^+ \rightarrow p\bar{p}$ or $\Lambda\bar{\Lambda}$ are governed by two form factors $G_E(P^2)$ and $G_M(P^2)$, with time-like argument P^2 . The unpolarized distribution is proportional to a certain linear combination of $|G_E|^2$ and $|G_M|^2$. The combination $\text{Im}(G_M G_E^*)$ is proportional to the only non-vanishing component of the vector polarization P_y . An independent linear combination of $|G_E|^2$ and $|G_M|^2$ can be obtained from any of the diagonal components of the tensor polarization A_{ii} . The only non-vanishing non-diagonal components of the tensor polarization, $A_{xz} = A_{zx}$ are proportional to $\text{Re}(G_M G_E^*)$. For a complete determination of the form factors these four quantities must be measured.

The reactions $e^-e^+ \rightarrow p\bar{p}$ and $\Lambda\bar{\Lambda}$ are presently under investigation at BESIII, the Beijing Spectrometer III. Details of the analysis and measurement of such reactions are given in the Varenna lectures of Johansson [1]. Expressions for the vector and tensor polarizations were first calculated by Dubničkova *et al.* [2]. Unfortunately, this reference has many misprints. Corrected expressions are given by Gakh and Tomasi-Gustafsson [3]. The expressions of these authors are confirmed by Buttimore and Jennings [4]. The reason for writing this paper is that we believe there are still errors in the last two publications. In contrast to their conclusion we claim that also the non-diagonal components $A_{yx} = A_{xy}$ vanish, leaving $A_{xz} = A_{zx}$ as the only non-vanishing non-diagonal components.

In addition to calculating the vector and tensor polarizations we calculate the hyperon-decay distributions in the reaction $e^-e^+ \rightarrow \Lambda(\rightarrow p\pi^-)\bar{\Lambda}(\rightarrow \bar{p}\pi^+)$. For this purpose we use the folding method of Czyż *et al.* [5], but in the covariant version developed in ref. [6], with proper counting of intermediate states. This application requires knowledge of the cross-section distributions for $e^-e^+ \rightarrow \Lambda\bar{\Lambda}$ with polarized hyperons.

II. LAMBDA FORM FACTORS

The reaction under consideration is described by the diagram of fig.1. Momentum definitions are also indicated there. The couplings of the initial state leptons are determined by the electron charge. Form factors or anomalous magnetic moments are not considered.

In the current matrix elements of the lambda, however, both form factors are taken into

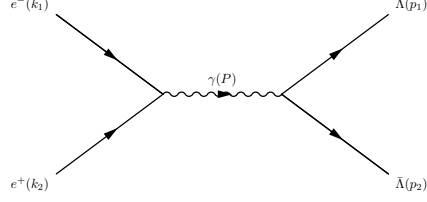


FIG. 1: Graph describing the reaction $e^+e^- \rightarrow \bar{\Lambda}\Lambda$.

account. We follow common practice [6] and write the hadron current matrix element as

$$j_\mu(p_1, p_2) = -ie\bar{u}(p_1)O_\mu(p_1, p_2)v(p_2), \quad (1)$$

$$O_\mu(p_1, p_2) = G_1(P^2)\gamma_\mu - \frac{1}{2M}G_2(P^2)Q_\mu, \quad (2)$$

with $P = p_1 + p_2$ and $Q = p_1 - p_2$. The lambda mass is denoted M .

The form factors G_1 and G_2 are related to the more commonly used form factors F_1 and F_2 , and the electric G_E and magnetic G_M form factors [5, 7, 8], through

$$G_1 = F_1 + F_2 = G_M \quad (3)$$

$$G_2 = F_2 = \frac{1}{1+\tau}(G_M - G_E) = \frac{4M^2}{Q^2}(G_M - G_E), \quad (4)$$

and $\tau = -P^2/4M^2$. The arguments of the form factors are all equal to P^2 . In particular, when $P^2 = 4M^2$ then $G_M = G_E = F_1 + F_2$. Another useful relation is

$$Q^2 = -4p^2, \quad (5)$$

where p is the final-state c.m. momentum.

III. CROSS SECTION

We follow Pilkuhn [7] and write the cross-section distribution of the reaction $e^+e^- \rightarrow \bar{\Lambda}\Lambda$ as

$$d\sigma = \frac{1}{2\sqrt{\lambda(s, m_e^2, m_e^2)}} \overline{|\mathcal{M}|^2} d\text{Lips}(k_1 + k_2; p_1, p_2), \quad (6)$$

where the average over the squared matrix element indicates average over initial electron and positron spins. The definitions of the particle momenta follows fig. 1.

We remove some trivial factors from the squared matrix element, namely the powers of the electron charge and the square of the intermediate-photon denominator, and write

$$\overline{|\mathcal{M}|^2} = \left(\frac{e^2}{P^2}\right)^2 \overline{|\mathcal{M}_{red}|^2} . \quad (7)$$

The reduced matrix element is decomposed as

$$\overline{|\mathcal{M}_{red}|^2} = L_{\nu\mu} K^{\nu\mu} , \quad (8)$$

where $L_{\nu\mu}$ and $K_{\nu\mu}$ are the lepton and hadron electromagnetic tensors.

IV. LEPTON TENSOR

The lepton tensor is by definition equal to

$$L_{\nu\mu}(k_1, k_2) = \frac{1}{4} \text{Sp}[\gamma_\nu(\not{k}_2 - m_e)\gamma_\mu(\not{k}_1 + m_e)] , \quad (9)$$

and takes care of the average over initial-state-lepton spins. We shall neglect the electron mass m_e compared with other masses and energies. In this approximation

$$\begin{aligned} L_{\nu\mu} &= L_{\mu\nu} \\ &= k_{1\nu}k_{2\mu} + k_{2\nu}k_{1\mu} - \frac{1}{2}sg_{\nu\mu}, \end{aligned} \quad (10)$$

with

$$s = (k_1 + k_2)^2 = P^2. \quad (11)$$

The lepton tensor enters the cross-section distribution contracted with the hadron tensor. The hadron tensor is gauge invariant, which means that when contracted with four vectors P^μ or P^ν zero result is obtained. Hence, dependencies P_μ or P_ν in the lepton tensor may be ignored, and as an example, we may replace eq. (10) by

$$L_{\nu\mu} = -2k_{1\nu}k_{1\mu} - \frac{1}{2}sg_{\nu\mu} . \quad (12)$$

V. HADRON TENSOR

The hadron tensor is calculated in Appendix B of ref [6]. We simply copy the formulae given there. Starting from the hadron vertex $O_\mu(p_1, p_2)$ of eq. (2) we get

$$\begin{aligned} K_{\nu\mu}(s_1, s_2) &= \text{Sp}[\bar{O}_\nu(\not{p}_1 + M)\frac{1}{2}(1 + \gamma_5 \not{s}_1) \\ &\quad \times O_\mu(\not{p}_2 - M)\frac{1}{2}(1 + \gamma_5 \not{s}_2)], \end{aligned} \quad (13)$$

with $\bar{O} = \gamma_0 O^\dagger \gamma_0$. The four vectors s_1 and s_2 denote the spin vectors of hyperon and anti-hyperon.

The hadronic tensor is gauge invariant, i.e. vanishes when contracted by P^ν or P^μ , and is decomposed as

$$K_{\nu\mu}(s_1, s_2) = K_{\nu\mu}^{00}(0, 0) + K_{\nu\mu}^{05}(s_1, 0) + K_{\nu\mu}^{50}(0, s_2) + K_{\nu\mu}^{55}(s_1, s_2). \quad (14)$$

The functional arguments indicate the spin vectors involved.

The explicit expression for the first part of the hadron tensor is

$$K_{\nu\mu}^{00} = \frac{1}{2} \left(P_\nu P_\mu - P^2 g_{\nu\mu} - Q_\nu Q_\mu \right) |G_1|^2 + \frac{1}{2} Q_\nu Q_\mu \left(2\text{Re}(G_1 G_2^*) - \frac{Q^2}{4M^2} |G_2|^2 \right). \quad (15)$$

Since the lepton tensor is symmetric in its indices we need only retain the symmetric parts of the hadron tensor. It follows that

$$K_{\nu\mu}^{05}(s_1, 0) = \frac{-1}{2M} \text{Im}(G_1 G_2^*) \left[Q_\nu \epsilon(p_1, p_2, s_1)_\mu + Q_\mu \epsilon(p_1, p_2, s_1)_\nu \right], \quad (16)$$

$$K_{\nu\mu}^{50}(0, s_2) = \frac{-1}{2M} \text{Im}(G_1 G_2^*) \left[Q_\nu \epsilon(p_1, p_2, s_2)_\mu + Q_\mu \epsilon(p_1, p_2, s_2)_\nu \right], \quad (17)$$

with the epsilon-function combinations defined as

$$\epsilon(p_2 p_1 l_1)_\nu = \epsilon_{\alpha\beta\gamma\nu} p_2^\alpha p_1^\beta l_1^\gamma, \quad (18)$$

$$\epsilon(p_2 p_1)_{\nu\mu} = \epsilon_{\alpha\beta\nu\mu} p_2^\alpha p_1^\beta, \quad (19)$$

and $\epsilon_{0123} = 1$.

The contribution depending on both spin vectors is

$$K_{\nu\mu}^{55}(s_1, s_2) = |G_1|^2 B_{\nu\mu}^1 + |G_2|^2 B_{\nu\mu}^2 + \text{Re}(G_1 G_2^*) B_{\nu\mu}^3, \quad (20)$$

with

$$\begin{aligned}
B_{\nu\mu}^1 = & -s_1 \cdot s_2 \left[p_{1\nu} p_{2\mu} + p_{1\mu} p_{2\nu} - \frac{1}{2} g_{\nu\mu} P^2 \right] \\
& - \frac{1}{2} P^2 (s_{1\nu} s_{2\mu} + s_{1\mu} s_{2\nu}) - g_{\nu\mu} p_1 \cdot s_2 p_2 \cdot s_1 \\
& + p_1 \cdot s_2 (s_{1\nu} p_{2\mu} + s_{1\mu} p_{2\nu}) \\
& + p_2 \cdot s_1 (s_{2\nu} p_{1\mu} + s_{2\mu} p_{1\nu}), \tag{21}
\end{aligned}$$

$$B_{\nu\mu}^2 = \frac{1}{4M^2} Q_\nu Q_\mu \left[\frac{1}{2} Q^2 s_1 \cdot s_2 + p_1 \cdot s_2 p_2 \cdot s_1 \right], \tag{22}$$

$$\begin{aligned}
B_{\nu\mu}^3 = & -Q_\nu Q_\mu s_1 \cdot s_2 + \frac{1}{2} \left[p_1 \cdot s_2 (Q_\nu s_{1\mu} + Q_\mu s_{1\nu}) \right. \\
& \left. - p_2 \cdot s_1 (Q_\nu s_{2\mu} + Q_\mu s_{2\nu}) \right]. \tag{23}
\end{aligned}$$

VI. POLARIZATION VARIABLES

Our polarization variables \mathcal{P}^{ab} are defined as follows. For unpolarized final-hadron states the factor $|\overline{\mathcal{M}_{red}}|^2$ of eq. (8) becomes

$$\begin{aligned}
\mathcal{P}^{00} &= \sum_{\pm s_1, \pm s_2} L \cdot K(s_1, s_2) \\
&= 4L \cdot K^{00}(0, 0). \tag{24}
\end{aligned}$$

Correspondingly, for polarized hyperon and unpolarized anti-hyperon

$$\begin{aligned}
\mathcal{P}^{05} &= \sum_{\pm s_2} L \cdot K(s_1, s_2) - L \cdot K(-s_1, s_2) \\
&= 4L \cdot K^{05}(s_1, 0), \tag{25}
\end{aligned}$$

whereas for unpolarized hyperon and polarized anti-hyperon

$$\begin{aligned}
\mathcal{P}^{50} &= \sum_{\pm s_1} L \cdot K(s_1, s_2) - L \cdot K(s_1, -s_2) \\
&= 4L \cdot K^{50}(0, s_2). \tag{26}
\end{aligned}$$

Finally, in the joint- or tensor-polarization case

$$\begin{aligned}
\mathcal{P}^{55} &= L \cdot K(s_1, s_2) - L \cdot K(s_1, -s_2) \\
&\quad + L \cdot K(-s_1, -s_2) - L \cdot K(-s_1, s_2) \\
&= 4L \cdot K^{55}(s_1, s_2). \tag{27}
\end{aligned}$$

The spin four-vector $s = s(p, n)$ satisfies $s \cdot s = -1$ and $s \cdot p = 0$, where p is the hadron four-momentum. The mass of the hadron is M , and its spin vector $-s(p, n)$ for polarisation $-\mathbf{n}$.

In the rest system of the hadron $s(p, \mathbf{n}) = (0, \mathbf{n})$ and $p = (M, \mathbf{0})$. In a coordinate system where the hadron has three-momentum \mathbf{p} , the spin vector is

$$s(\mathbf{p}, \mathbf{n}) = \frac{n_{\parallel}}{M}(|\mathbf{p}|, E\hat{\mathbf{p}}) + (0, \mathbf{n}_{\perp}), \quad (28)$$

with $n_{\parallel} = \mathbf{n} \cdot \hat{\mathbf{p}}$ and

$$\mathbf{n}_{\perp} = \mathbf{n} - \hat{\mathbf{p}}(\mathbf{n} \cdot \hat{\mathbf{p}}). \quad (29)$$

In the first part of expression (28) we notice the helicity vector $h(p) = (|\mathbf{p}|, E\hat{\mathbf{p}})/M$.

We have evaluated these polarization variables in the global c.m. system, where

$$\mathbf{p}_1 = -\mathbf{p}_2 = \mathbf{p}, \quad (30)$$

$$\mathbf{k}_1 = -\mathbf{k}_2 = \mathbf{k}, \quad (31)$$

and with scattering angle

$$\cos(\theta) = \hat{\mathbf{p}} \cdot \hat{\mathbf{k}}. \quad (32)$$

The polarization vectors of the final-state hadrons are \mathbf{n}_1 and \mathbf{n}_2 in their respective rest systems (cf eq. (28)).

It is convenient to express the polarization variables in terms of the form-factor combinations

$$D_c = 2s|G_M|^2, \quad (33)$$

$$D_s = s|G_M|^2 - 4M^2|G_E|^2. \quad (34)$$

The unpolarized variable becomes

$$\mathcal{P}^{00} = s(D_c - D_s \sin^2(\theta)), \quad (35)$$

and the vector polarization

$$\mathcal{P}^{05} = \mathcal{S} \left[\frac{1}{\sin(\theta)} (\hat{\mathbf{p}} \times \hat{\mathbf{k}}) \cdot \mathbf{n}_1 \right], \quad (36)$$

with the function \mathcal{S} defined as

$$\mathcal{S} = -2Ms\sqrt{s} \sin(2\theta) \text{Im}(G_M G_E^*). \quad (37)$$

To get \mathcal{P}^{50} from (36) we replace \mathbf{n}_1 by \mathbf{n}_2 , the variable \mathcal{S} remaining the same,

$$\mathcal{P}^{50} = \mathcal{S} \left[\frac{1}{\sin(\theta)} (\hat{\mathbf{p}} \times \hat{\mathbf{k}}) \cdot \mathbf{n}_2 \right]. \quad (38)$$

The joint or tensor polarization, finally, can be written as

$$\begin{aligned} \mathcal{P}^{55} = & \mathcal{T}_1 \mathbf{n}_1 \cdot \hat{\mathbf{p}} \mathbf{n}_2 \cdot \hat{\mathbf{p}} + \mathcal{T}_2 \mathbf{n}_{1\perp} \cdot \mathbf{n}_{2\perp} + \mathcal{T}_3 \mathbf{n}_{1\perp} \cdot \hat{\mathbf{k}} \mathbf{n}_{2\perp} \cdot \hat{\mathbf{k}} \\ & + \mathcal{T}_4 \left\{ \mathbf{n}_1 \cdot \hat{\mathbf{p}} \mathbf{n}_{2\perp} \cdot \hat{\mathbf{k}} + \mathbf{n}_2 \cdot \hat{\mathbf{p}} \mathbf{n}_{1\perp} \cdot \hat{\mathbf{k}} \right\}, \end{aligned} \quad (39)$$

with functions

$$\mathcal{T}_1 = s [D_s \sin^2(\theta) + D_c \cos^2(\theta)] \quad (40)$$

$$\mathcal{T}_2 = -s D_s \sin^2(\theta) \quad (41)$$

$$\mathcal{T}_3 = s D_c \quad (42)$$

$$\mathcal{T}_4 = 4Ms\sqrt{s} \operatorname{Re}(G_M G_E^*) \cos(\theta). \quad (43)$$

Note that \mathcal{P}^{55} is symmetric in \mathbf{n}_1 and \mathbf{n}_2 .

VII. CARTESIAN OBSERVABLES

Before discussing the formulae above we introduce an ortho-normalized system of coordinates. The scattering plane with the vectors \mathbf{p} and \mathbf{k} make up the xz -plane, with the y -axis along the normal to the scattering plane,

$$\mathbf{e}_z = \hat{\mathbf{p}}, \quad (44)$$

$$\mathbf{e}_y = \frac{1}{\sin(\theta)} (\hat{\mathbf{p}} \times \hat{\mathbf{k}}), \quad (45)$$

$$\mathbf{e}_x = \frac{1}{\sin(\theta)} (\hat{\mathbf{p}} \times \hat{\mathbf{k}}) \times \hat{\mathbf{p}}. \quad (46)$$

Expressed in terms of these basis vectors

$$\hat{\mathbf{k}} = \sin(\theta) \mathbf{e}_x + \cos(\theta) \mathbf{e}_z. \quad (47)$$

We first calculate the unpolarized differential cross-section distribution. Combining eqs (6-8) and (35) we get

$$\begin{aligned} \frac{d\sigma}{d\Omega} &= \frac{\alpha^2}{4s^3} \frac{p}{k} \mathcal{P}^{00} = \frac{\alpha^2}{4s^3} \frac{p}{k} (\mathcal{T}_2 + \mathcal{T}_3) \\ &= \frac{\alpha^2}{4s^2} \frac{p}{k} (D_c - D_s \sin^2(\theta)). \end{aligned} \quad (48)$$

The components of the vector-polarization are obtained from eq. (36) by successively putting the polarization vector \mathbf{n}_1 equal to \mathbf{e}_x , \mathbf{e}_y , and \mathbf{e}_z ,

$$\mathcal{P}_y^{05} = \mathcal{S} = -2Ms\sqrt{s}\sin(2\theta)\text{Im}(G_M G_E^*), \quad (49)$$

$$\mathcal{P}_x^{05} = \mathcal{P}_z^{05} = 0. \quad (50)$$

The variable \mathcal{P}_i^{05} is related to the more commonly used normalized-vector polarization through $P_i = \mathcal{P}_i^{05}/\mathcal{P}^{00}$. For the anti-hyperons the polarization components are the same as for hyperons.

The components of the tensor polarizations are obtained from eq. (39). The diagonal components are

$$\mathcal{P}_{yy}^{55} = \mathcal{T}_2 = -sD_s \sin^2(\theta), \quad (51)$$

$$\mathcal{P}_{xx}^{55} = \mathcal{T}_3 - \mathcal{T}_1 = s[D_c - D_s] \sin^2(\theta), \quad (52)$$

$$\mathcal{P}_{zz}^{55} = \mathcal{T}_1 = s[D_s \sin^2(\theta) + D_c \cos^2(\theta)], \quad (53)$$

the non-vanishing non-diagonal components

$$\begin{aligned} \mathcal{P}_{xz}^{55} &= \sin(\theta)\mathcal{T}_4 = 2Ms\sqrt{s}\sin(2\theta)\text{Re}(G_M G_E^*), \\ &= \mathcal{P}_{zx}^{55}, \end{aligned} \quad (54)$$

and the vanishing non-diagonal components

$$\mathcal{P}_{xy}^{55} = \mathcal{P}_{yx}^{55} = \mathcal{P}_{zy}^{55} = \mathcal{P}_{yz}^{55} = 0. \quad (55)$$

The commonly employed normalized-joint polarizations are related to the above ones by $A_{ij} = \mathcal{P}_{ij}^{55}/\mathcal{P}^{00}$.

In the tensor polarizations the first index refers to the anti-hyperon (particle 2) and the second to the hyperon (particle 1). However, because of the symmetry the remark is irrelevant.

The polarizations and joint polarizations have been calculated before. We have three articles in mind. The first one [2] contains several misprints. In the second one [3] errors are corrected. However, both articles suggest a non-vanishing value for A_{xy} , in disagreement with our result in eq. (55). For the remaining vector and tensor polarizations we agree with ref. [3]. Note that our choice of coordinate system differs from that of [3]. The third article [4] claims to obtain the same result as that of ref. [3]. However, it is easy to understand that if A_{zy} vanishes, then by symmetry the same must be true for A_{xy} .

VIII. HYPERON DECAY DISTRIBUTIONS

So far the analysis is valid for $e^+e^- \rightarrow \bar{\Lambda}\Lambda$ as well as for $e^+e^- \rightarrow \bar{p}p$, but with different hadron form factors. Now, we restrict ourselves to the first reaction.

The decay distributions are obtained via the folding method of refs [6] and [5]. In this approach the hyperon-production distributions are multiplied by the hyperon-decay distributions and averaged over the intermediate hyperon-spin directions. A factor of four must also be added since there are four spin combinations, as pointed out in ref. [6]. The labelling of momenta are explained in fig. 2.

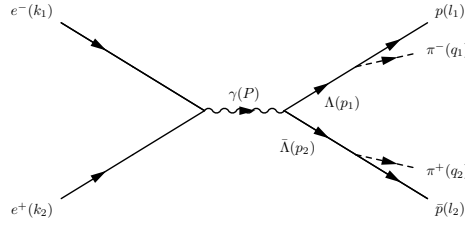


FIG. 2: Graph describing the reaction $e^+e^- \rightarrow \Lambda(\rightarrow p\pi^-)\bar{\Lambda}(\rightarrow \bar{p}\pi^+)$.

According to the folding hypothesis the distribution function is, when summed over final hadron spins,

$$\begin{aligned} \overline{|\mathcal{M}|^2} &= \sum_{\pm s_1, \pm s_2} \left\langle |\mathcal{M}(e^+e^- \rightarrow \Lambda(s_1)\bar{\Lambda}(s_2))|^2 \right. \\ &\quad \times \left. |\mathcal{M}(\Lambda(s_1) \rightarrow p\pi^-)|^2 |\mathcal{M}(\bar{\Lambda}(s_2) \rightarrow \bar{p}\pi^+)|^2 \right\rangle_{\mathbf{n}_1 \mathbf{n}_2}, \end{aligned} \quad (56)$$

with the production distribution

$$|\mathcal{M}(e^+e^- \rightarrow \Lambda(s_1)\bar{\Lambda}(s_2))|^2 = L \cdot K(s_1, s_2), \quad (57)$$

and the decay distribution, summed over proton spins,

$$\begin{aligned} |\mathcal{M}(\Lambda(s_1) \rightarrow p\pi^-)|^2 &= R_\Lambda [1 - \alpha_\Lambda l_1 \cdot s_1 / l_\Lambda] \\ &= R_\Lambda \left[1 + \alpha_\Lambda \mathbf{n}_1 \cdot \hat{\mathbf{l}}_{1c} \right]. \end{aligned} \quad (58)$$

Here, \mathbf{l}_{1c} is the proton momentum in the hyperon-rest system, local c.m. system, and of length $|\mathbf{l}_{1c}| = l_\Lambda$. Expressed in terms of the proton momentum in the global c.m. system,

$$\mathbf{l}_{1c\perp} = \mathbf{l}_{1\perp}, \quad (59)$$

$$l_{1c\parallel} = \mathbf{l}_{1c} \cdot \hat{\mathbf{p}} = \frac{1}{p} [ME_1 - EE_{1c}]. \quad (60)$$

The proton energy is E_{1c} in the hyperon-rest system and E_1 in the global c.m. system where the hyperon momentum is \mathbf{p} . The corresponding hyperon energies are M and E .

The decay distribution for the anti-hyperon is similarly

$$\begin{aligned} |\mathcal{M}(\bar{\Lambda}(s_2) \rightarrow \bar{p}\pi^+)|^2 &= R_\Lambda [1 + \alpha_\Lambda l_2 \cdot s_2 / l_\Lambda] \\ &= R_\Lambda [1 - \alpha_\Lambda \mathbf{n}_2 \cdot \hat{\mathbf{l}}_{2c}], \end{aligned} \quad (61)$$

with

$$\mathbf{l}_{2c\perp} = \mathbf{l}_{2\perp}, \quad (62)$$

$$l_{2c\parallel} = \mathbf{l}_{2c} \cdot \hat{\mathbf{p}} = -\frac{1}{p} [ME_2 - EE_{2c}]. \quad (63)$$

Here, we have used the fact that $\alpha_{\bar{\Lambda}} = -\alpha_\Lambda$ and that the anti-hyperon has momentum $-\mathbf{p}$ in the global c.m. system.

The sum over the spin components in eq. (56) can be replaced by a factor of 4 since each spin combination gives the same result when averaged over spin directions \mathbf{n}_1 and \mathbf{n}_2 .

The averages over the spin vectors \mathbf{n}_1 and \mathbf{n}_2 in eq. (56) are easily carried out since

$$\langle (\mathbf{n} \cdot \mathbf{l}) \mathbf{n} \rangle_{\mathbf{n}} = \mathbf{l}. \quad (64)$$

The decay distribution is decomposed as,

$$|\overline{\mathcal{M}}|^2 = R_\Lambda^2 [G^{00} + G^{05} + G^{50} + G^{55}], \quad (65)$$

where R_Λ is related to the decay rate of the lambda hyperon (Appendix A of [6]). The G functions are

$$G^{00} = \mathcal{P}^{00} = \mathcal{T}_2 + \mathcal{T}_3, \quad (66)$$

$$G^{05} = \alpha_\Lambda \mathcal{S} \left[\frac{1}{\sin(\theta)} (\hat{\mathbf{p}} \times \hat{\mathbf{k}}) \cdot \hat{\mathbf{l}}_{1c} \right], \quad (67)$$

$$G^{50} = \alpha_\Lambda (-\mathcal{S}) \left[\frac{1}{\sin(\theta)} (\hat{\mathbf{p}} \times \hat{\mathbf{k}}) \cdot \hat{\mathbf{l}}_{2c} \right], \quad (68)$$

$$\begin{aligned} G^{55} = \alpha_\Lambda^2 \Big\{ & -\mathcal{T}_1 \hat{\mathbf{l}}_{1c} \cdot \hat{\mathbf{p}} \hat{\mathbf{l}}_{2c} \cdot \hat{\mathbf{p}} - \mathcal{T}_2 \hat{\mathbf{l}}_{1c\perp} \cdot \hat{\mathbf{l}}_{2c\perp} \\ & - \mathcal{T}_3 \hat{\mathbf{l}}_{1c\perp} \cdot \hat{\mathbf{k}} \hat{\mathbf{l}}_{2c\perp} \cdot \hat{\mathbf{k}} \\ & - \mathcal{T}_4 \left(\hat{\mathbf{l}}_{1c} \cdot \hat{\mathbf{p}} \hat{\mathbf{l}}_{2c\perp} \cdot \hat{\mathbf{k}} + \hat{\mathbf{l}}_{2c} \cdot \hat{\mathbf{p}} \hat{\mathbf{l}}_{1c\perp} \cdot \hat{\mathbf{k}} \right) \Big\}. \end{aligned} \quad (69)$$

These equations are expressed in terms of the proton and anti-proton momenta in the hyperon and anti-hyperon rest systems. However, it should be noted that $\mathbf{l}_{ic\perp} = \mathbf{l}_{i\perp}$, for $i = 1, 2$.

The normalization of the cross-section distribution is as follows

$$\begin{aligned} \frac{d\sigma}{d\Omega_\Lambda} &= \frac{1}{64\pi^2 s} \frac{p}{k} \left(\frac{4\pi\alpha}{s} \right)^2 \frac{\Gamma_\Lambda \Gamma_{\bar{\Lambda}}}{\Gamma^2(M)} \left(\sum_{a,b} G^{ab} \right) \\ &\times \left[\frac{d\Omega_p}{4\pi} \right]_\Lambda \left[\frac{d\Omega_{\bar{p}}}{4\pi} \right]_{\bar{\Lambda}}. \end{aligned} \quad (70)$$

Here, k and p are the initial- and final-state momenta in the reaction $e^+e^- \rightarrow \Lambda\bar{\Lambda}$. The angles Ω_p and $\Omega_{\bar{p}}$ are angles measured in the rest systems of Λ and $\bar{\Lambda}$, and Ω_Λ in the global c.m system. Γ_Λ and $\Gamma_{\bar{\Lambda}}$ are the channel widths,

If we integrate over the lambda-decay angles Ω_p then the contributions from the functions G^{05} and G^{55} are annulled [6], and correspondingly for the angles $\Omega_{\bar{p}}$.

IX. EXTRACTING FORM FACTORS

The coefficients appearing in eqs (66-69) are orthogonal when integrated over the decay distributions of the hyperons, a property which makes it easy to extract the \mathcal{S} and \mathcal{T} functions from the cross-section distributions.

We first make the definitions

$$G = G^{00} + G^{05} + G^{50} + G^{55}, \quad (71)$$

$$d\omega = \left[\frac{d\Omega_p}{4\pi} \right]_\Lambda \left[\frac{d\Omega_{\bar{p}}}{4\pi} \right]_{\bar{\Lambda}}. \quad (72)$$

With suitable weight functions we get the following extraction formulas

$$\int d\omega G = \mathcal{P}^{00} = \mathcal{T}_2 + \mathcal{T}_3, \quad (73)$$

$$\int d\omega G \mathbf{e}_y \cdot \hat{\mathbf{l}}_{1c} = \frac{1}{3} \alpha_\Lambda \mathcal{S}, \quad (74)$$

$$\int d\omega G \mathbf{e}_y \cdot \hat{\mathbf{l}}_{2c} = -\frac{1}{3} \alpha_\Lambda \mathcal{S}, \quad (75)$$

$$\int d\omega G \hat{\mathbf{l}}_{1c} \cdot \hat{\mathbf{p}} \hat{\mathbf{l}}_{2c} \cdot \hat{\mathbf{p}} = -\frac{1}{9} \alpha_\Lambda^2 \mathcal{T}_1, \quad (76)$$

$$\int d\omega G \hat{\mathbf{l}}_{1c\perp} \cdot \hat{\mathbf{l}}_{2c\perp} = -\frac{2}{9} \alpha_\Lambda^2 \mathcal{T}_2, \quad (77)$$

$$\int d\omega G \hat{\mathbf{l}}_{1c\perp} \cdot \hat{\mathbf{k}} \hat{\mathbf{l}}_{2c\perp} \cdot \hat{\mathbf{k}} = -\frac{1}{9} \alpha_\Lambda^2 \sin^4(\theta) \mathcal{T}_3, \quad (78)$$

$$\int d\omega G \hat{\mathbf{l}}_{1c} \cdot \hat{\mathbf{p}} \hat{\mathbf{l}}_{2c\perp} \cdot \hat{\mathbf{k}} = -\frac{1}{9} \alpha_\Lambda^2 \sin^2(\theta) \mathcal{T}_4. \quad (79)$$

We conclude that the \mathcal{S} and \mathcal{T} functions can be measured either via the vector and tensor polarizations as discussed in Sect. VII, or by integrating the decay-angular distributions with deftly chosen weight functions.

We first integrate the measured cross section distribution G over the decay distributions of both hyperons, eq.(73), to get

$$\int d\omega G = \mathcal{T}_2 + \mathcal{T}_3 = D_c \left[1 - \frac{D_s}{D_c} \sin^2(\theta) \right]. \quad (80)$$

By analyzing the angular dependence of this distribution we determine the ratio D_s/D_c , which in turn yields the ratio of the norms of the form factors $|G_M|/|G_E|$. If the absolute normalization of the cross section can be measured as well we get in addition the norms of the individual form factors $|G_M|$ and $|G_E|$.

After integration over both hyperon decay distributions, eq.(70) becomes the cross section distribution for the reaction $e^+e^- \rightarrow \Lambda \bar{\Lambda}$, eq.(48), but with the extra probability factor describing the simultaneous decays $\Lambda \rightarrow p\pi^-$ and $\bar{\Lambda} \rightarrow \bar{p}\pi^+$,

$$\Gamma_\Lambda(\Lambda \rightarrow p\pi^-) \Gamma_{\bar{\Lambda}}(\bar{\Lambda} \rightarrow \bar{p}\pi^+) / \Gamma^2(M).$$

This is certainly reassuring.

The structure function called \mathcal{S} contains as a factor $\text{Im}(G_M G_E^*)$. Consequently, this factor can be determined by integrating the measured cross section distribution with the projector

of eq.(74),

$$\begin{aligned}\int d\omega G \mathbf{e}_y \cdot \hat{\mathbf{l}}_{1c} &= \frac{1}{3} \alpha_\Lambda \mathcal{S} \\ &= -\frac{2}{3} \alpha_\Lambda M s \sqrt{s} \sin(2\theta) \text{Im}(G_M G_E^*),\end{aligned}\tag{81}$$

with \mathcal{S} from eq.(37).

In order to determine the form factors completely we also need the combination $\text{Re}(G_M G_E^*)$, which is a factor in \mathcal{T}_4 of eq.(43). This structure function is obtained with the projector of eq.(79). Therefore,

$$\begin{aligned}\int d\omega G \hat{\mathbf{l}}_{1c} \cdot \hat{\mathbf{p}} \hat{\mathbf{l}}_{2c\perp} \cdot \hat{\mathbf{k}} &= -\frac{1}{9} \alpha_\Lambda^2 \sin^2(\theta) \mathcal{T}_4 \\ &= -\frac{2}{9} \alpha_\Lambda^2 M s \sqrt{s} \cos(\theta) \sin(2\theta) \text{Re}(G_M G_E^*).\end{aligned}\tag{82}$$

It might be appropriate to remark that if the norms of the form factors and the imaginary part as above are known then only the sign of the real part is free. It is determined by eq.(82).

There are four additional projections to be made. They will not give new information but may strengthen results already obtained.

Appendix A: Covariant distributions

The expressions involving the hyperon decays, eqs (66-69), can be made explicitly covariant by following the methods in [6].

From eqs (25) and (26) we get

$$\begin{aligned}\mathcal{P}^{05}(s_1, 0) &= \frac{4M}{p^2} \text{Im}(G_M G_E^*) \\ &\quad \times (k_1 - k_2) \cdot Q \epsilon(p_1, p_2, s_1, k_1),\end{aligned}\tag{A1}$$

$$\begin{aligned}\mathcal{P}^{50}(0, s_2) &= \frac{4M}{p^2} \text{Im}(G_M G_E^*) \\ &\quad \times (k_2 - k_1) \cdot Q \epsilon(p_1, p_2, s_2, k_2).\end{aligned}\tag{A2}$$

It should be observed that

$$\epsilon(p_1, p_2, s_2, k_2) = -\epsilon(p_1, p_2, s_2, k_1).\tag{A3}$$

The folding procedure of eq. (56) leads to

$$G^{05} = -\frac{\alpha_\Lambda}{l_\Lambda} \left\langle l_1 \cdot s_1 \mathcal{P}^{05} \right\rangle_{\mathbf{n}_1}, \quad (\text{A4})$$

$$G^{50} = \frac{\alpha_\Lambda}{l_\Lambda} \left\langle l_2 \cdot s_2 \mathcal{P}^{50} \right\rangle_{\mathbf{n}_2}. \quad (\text{A5})$$

The average over the directions \mathbf{n} of the spin vector $s(p, \mathbf{n})$ is accomplished by

$$\langle s^\mu(p, \mathbf{n}) s^\nu(p, \mathbf{n}) \rangle_{\mathbf{n}} = \frac{1}{M^2} p^\mu p^\nu - g^{\mu\nu}, \quad (\text{A6})$$

which in turn leads to

$$\langle (s \cdot l) s \rangle_{\mathbf{n}} = \lambda p - l, \quad (\text{A7})$$

$$\begin{aligned} \lambda &= \frac{p \cdot l}{M^2} = E_0/M \\ &= \frac{1}{2M^2} (M^2 + m^2 - \mu^2), \end{aligned} \quad (\text{A8})$$

where E_0 is the energy of the decay proton in the hyperon rest system.

This gives the results

$$\begin{aligned} G^{05} &= \frac{\alpha_\Lambda}{l_\Lambda} \frac{4M}{p^2} \text{Im}(G_M G_E^*) \\ &\quad \times (k_1 - k_2) \cdot Q \epsilon(p_1, p_2, l_1, k_1), \end{aligned} \quad (\text{A9})$$

$$\begin{aligned} G^{50} &= -\frac{\alpha_\Lambda}{l_\Lambda} \frac{4M}{p^2} \text{Im}(G_M G_E^*) \\ &\quad \times (k_2 - k_1) \cdot Q \epsilon(p_1, p_2, l_2, k_2). \end{aligned} \quad (\text{A10})$$

These two equations are identical to those of eqs (67) and (68).

We start from eq. (27) which we expand as

$$\mathcal{P}^{55} = A_1 R_1 + A_2 R_2 + A_3 R_3 + A_4 R_4. \quad (\text{A11})$$

The A factors involve the following spin combinations

$$A_1 = s_1 \cdot s_2, \quad (\text{A12})$$

$$A_2 = P \cdot s_1 P \cdot s_2, \quad (\text{A13})$$

$$A_3 = q \cdot s_1 q \cdot s_2, \quad (\text{A14})$$

$$A_4 = P \cdot s_1 q \cdot s_2 - P \cdot s_2 q \cdot s_1, \quad (\text{A15})$$

with $q = k_1 - k_2$, and the R factors are calculated as

$$R_1 = s D_s \sin^2(\theta), \quad (\text{A16})$$

$$R_2 = \frac{1}{2p^2} \left[-s D_s \sin^2(\theta) - \frac{1}{2}(s + 4M^2) D_c \cos^2(\theta) + 8s M^2 \cos^2(\theta) \text{Re}(G_M G_E^*) \right], \quad (\text{A17})$$

$$R_3 = D_c, \quad (\text{A18})$$

$$R_4 = \frac{\sqrt{s}}{2p} \cos(\theta) \left[D_c - 8M^2 \text{Re}(G_M G_E^*) \right]. \quad (\text{A19})$$

We next return to the folding procedure of eq. (56). We do not sum over spin components but calculate for fixed spins, average over its directions, and multiply by the factor of four. The expression for the function G^{55} of eq. (65) reads

$$G^{55} = - \left(\frac{\alpha_\Lambda}{l_\Lambda} \right)^2 \left\langle l_1 \cdot s_1 l_2 \cdot s_2 \mathcal{P}^{55} \right\rangle_{\mathbf{n}_1 \mathbf{n}_2}. \quad (\text{A20})$$

Let us define

$$\bar{A}_i = \langle l_1 \cdot s_1 l_2 \cdot s_2 A_i \rangle_{\mathbf{n}_1 \mathbf{n}_2}. \quad (\text{A21})$$

for the four possible values of i . Then, a straightforward calculation yields

$$\begin{aligned} \bar{A}_1 &= (\lambda p_1 - l_1) \cdot (\lambda p_2 - l_2) \\ &= - \frac{2p^2 + M^2}{M^2} l_{1c\parallel} l_{2c\parallel} - \mathbf{l}_{1c\perp} \cdot \mathbf{l}_{2c\perp} \end{aligned} \quad (\text{A22})$$

$$\begin{aligned} \bar{A}_2 &= [\lambda p_1 \cdot P - P \cdot l_1] [\lambda p_2 \cdot P - P \cdot l_2] \\ &= - \frac{sp^2}{M^2} l_{1c\parallel} l_{2c\parallel}, \end{aligned} \quad (\text{A23})$$

$$\begin{aligned} \bar{A}_3 &= [\lambda p_1 \cdot q - q \cdot l_1] [\lambda p_2 \cdot q - q \cdot l_2] \\ &= s \left[\frac{\sqrt{s}}{2M} \cos(\theta) l_{1c\parallel} + \mathbf{l}_{1c\perp} \cdot \hat{\mathbf{k}} \right] \\ &\quad \times \left[\frac{\sqrt{s}}{2M} \cos(\theta) l_{2c\parallel} + \mathbf{l}_{2c\perp} \cdot \hat{\mathbf{k}} \right], \end{aligned} \quad (\text{A24})$$

$$\begin{aligned} \bar{A}_4 &= [\lambda p_1 \cdot P - P \cdot l_1] [\lambda p_2 \cdot q - q \cdot l_2] \\ &\quad - [\lambda p_2 \cdot P - P \cdot l_2] [\lambda p_1 \cdot q - q \cdot l_1] \\ &= - \frac{ps}{M} \left\{ l_{1c\parallel} \left[\frac{\sqrt{s}}{2M} \cos(\theta) l_{2c\parallel} + \mathbf{l}_{2c\perp} \cdot \hat{\mathbf{k}} \right] \right. \\ &\quad \left. + l_{2c\parallel} \left[\frac{\sqrt{s}}{2M} \cos(\theta) l_{1c\parallel} + \mathbf{l}_{1c\perp} \cdot \hat{\mathbf{k}} \right] \right\}. \end{aligned} \quad (\text{A25})$$

Inserting these results into \mathcal{P}^{55} of eq. (A11), and taking the average according to eq. (A20), gives the result reported in eq. (79).

Acknowledgments

Tord Johansson spotted the problem with the known tensor-polarization distributions, and suggested this investigation. I thank him, Andrzej, Karin, and Stefan for discussions.

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